UNREAL NUMBERS

This special character that the atom possesses is the appearance of *whole numbers.*

Max Born (1925)

No one thing ever merges gradually into anything else; the steps are discontinuous, but often so very minute as to seem truly continuous.

John Steinbeck (1941)

I tend more and more to the opinion that one cannot come further with a continuum theory.

Albert Einstein (1941)

"Gödel's Theorem," the professor said, sternly. "That damned nuisance."

Ross said nothing.

"Theorem states that certain proofs cannot be proved. Of course, he's right. That's the problem."

Michael Crichton (1969)

We should be trying to count something now or, put another way, we should be trying to formulate physics in a discontinuum rather than in a continuum.

Ted Jacobson (1988)

Geometry, as it appears in physics, might be a phenomenon that's "emergent" rather than fundamental.

Shing-Tung Yao (2010)

I'm buying a burrito from a vendor just off Victory when he again brings up the issue of how physics links to math. Walking back beneath a sinking sun and smoggy sky, I think about the piece I did on this for her detective. Though it wasn't much it was enough for me to see this as an old conundrum. It has a literature of its own; it's heavy stuff. Does he know this? If so, it seems that it won't deter him.

"The universe *is* math," his voice assures me. "It has arithmetic. It follows from the Rules."

It takes me half a block to see what he must mean. It starts with one. The Manifold. Or one Fleck, as we call them when we see there will be more. Move 1 adds another. One plus one is two. Or one could view this as multiplying. One times two is two. But until Move 1 there *is* no two yet in the universe. Just one. So it must be addition. Or maybe not: Two is also the successor, the next number after one. But Move 2 is not succeeding; it skips three. It is adding or *it* might be multiplying. Two plus two or two times two makes four. Multiplying soon looks simpler as the Fizzion hits its stride. Eight times two . . .

It dawns on me that he is thinking about Gödel's Incompleteness Theorem. Google helps me look it up again. Some say it's the high point of a century of math. It's 1931. Gödel proves that there are limits on what any system of math can prove. That is, every math system contains truths it cannot prove. The proof has a proviso: The math system must include all of the symbols used in arithmetic. 'These symbols are,' says Barrow, '0, "zero", S, "successor of", +, ×, and =.' What Frank is on about is . . .

"Where's the zero?"

It's not there. Or if it is, it's not in the Beginning and not in the Rules. S, +, × and = are present. But no zero. Someone might speculate that, in some sense, *before* the Beginning there were *zero* manifolds. But in what sense? Before goes with after. *After* Move 1 there are two Flecks. And before Move 1 just one. Before *that* makes no sense to me. For example: Before that there was no time. It's self-contradictory. The Beginning is Move 1 and is how time begins. There is nothing to be said about Time Zero. Maybe it can be thought about. But in the UC it does not compute. Like me instructing my computer: Print the state that you were in before the power came on. It knows not of it.

I can see where he is going. The UC is the most powerful computer that there ever was (or ever will be). Gödel's theorem seems to say that there are truths it cannot prove. But *his* idea is the theorem doesn't say this, because the universe's math does not have zero. Maybe he's right. Maybe too the theorem *would* apply if one took one step back from the Beginning. But the water there is far too deep for me.

Of more interest to me are limits the Beginning sets for physics. It says the universe is not continuous. It's starting to sink in how fundamental this may be. Physicists *assume* it is continuous. A secret society of Greek philosophers led by Pythagoras invents the concept. Their biggest secret is a kind of number, later called irrational, that can't be expressed as a fraction. They know the formula for the hypotenuse, the third side of a right-angled triangle. The simplest such triangle has sides of 1, 1 and √2. The square root of two is irrational; it's 1.414213562373… .

Rational numbers—fractions—have gaps between them. The irrational numbers fill the gaps. The two combined are known as real numbers. No gaps; they are complete. Real numbers allow physicists to see space as continuous. Time too. That doesn't mean they *are.* Though it's clear to me that this is fundamental to his story, he gives me no hint he understands. How to help him? Can we have a conversation? I try it with a simple game. I pick up a ruler. Its edge is a line. Will he see it? I think at him: Pick any two points on the line.

"One point five inches and eight point one inches."

He sees it! I make two paper pointers labeled Left and Right. Now he gets to play. I think: Pick a new point between Left and Right, not equal to either.

"Five point two."

I move Left to five point two. I think: This is a *Cut*. A number theorist called Dedekind gave it the name. Well, being German, he said *Schnitt*. It now provides the standard definition of real numbers.

I have him make another Cut. For no reason he picks 6.55 inches; Right. And another: 5.823 inches this time, Left again. Though he doesn't have to, he is picking roughly halfway every time.

Each Cut moves one pointer closer to the other. If space is continuous, he and I can keep on making Cuts forever—or until we tire. No matter how close Left gets to Right, he can always find a point between them. For example, a point that we can call New Left = (Left + Right)/2 must be between them. New Left is more than Left and less than Right. After an infinity of Cuts there will still be points remaining between Left and Right.

But, with space that's made of quantum pieces, now the game is different. Soon he finds New Left next to New Right. The Beginning says that in this universe at most 120 Cuts will butt New Left up hard against New Right and there *is no point between*. Indeed, there are no points. Taking this thought one step further, space that's made of Flecks allows no lines. Euclid would be baffled. The elements of his geometry do not exist.

Barrow says this kind of universe is far more complex than one that's continuous. That is, physics has adopted a simplistic view. It gives simplistic answers, which, though bad, may be convenient. But, worse, it asks simplistic questions. Physicists should focus on a universe that really is. It's such an irony: The real world doesn't run on real numbers! This has immense implications. Barrow says, 'Most fundamental pictures of the physical world assume that the basic notions—fields, space, and time—are continuous entities rather than discrete bits.' The fundamental pictures of the basic notions that he mentions are the so-called laws of physics. The Beginning says they all rest on assumptions that are incorrect. It says that physicists are barking up the wrong math tree. And Knuth has Conway—whom he made into his character—have Bill, Knuth's character, say the real numbers *are* the universe. By not addressing how things really work this is, in Pauli's phrase, not even wrong.

For starters, physics measures place in space with points and lines and distances. It's called position. His Beginning says it's not a thing to measure. There is really no such thing as a position. Pretending that there is creates a good approximation. There's no problem marking out a football field. Nor even finding atoms on a surface. But in dealing with the nature of reality it's a mistake. Julia Roberts gets it: 'Big mistake. Big. Huge.'

Anyone who's looking for a ToE in a continuum won't find it as it isn't there. Yet this is where most everyone who's looking's looking. And figuring positions. So, if not positions, what can they use that is real? He says volumes, counts them on his fingers, so to speak. Just 1, 2, 3, the numbers that he says the cosmos uses. Volume, he says, is the basic property of 3-D space. Its volume equals Fleck size times Fleck number and it can't be truly measured any other way. For example, I think of a cubic inch. Its sides must cut through Flecks. But it's clear from the Beginning that they can't be cut.

It's not just that we can't cut Flecks. To think of cutting misses the whole point. The difference between space that's continuous and space that's made of Flecks is that the ToE is built on properties of Flecks. The Beginning says that physicists can theorize about a single Fleck and the whole universe and anything between with the same set of rules.

"There are no axes." He sounds grumpy. He says axes as if rhymed with taxes. How does he know how to say a word?

I am sure that he means axes—rhymes with taxis—on a graph. Physicists use axes to show quantities like distance, say, or temperature or time. He means axes cut through Flecks so they are wrong. What then would he think of differential equations? They use axes and they are *the* math of physics. They describe the change in something over space and time. They are fine for practical approximations. But for anyone concerned with Fleck-scale physics and what really happens they are undefined and indefinable. It's not surprising physics is in trouble.

"Toast," he says, dismissively.

I take this to mean physics and its unreal math. Bold for a detective, one might think. Not to mention that whatever may be real he's surely not. If this notion catches his attention he's unfazed.

"General relativity is made of them."

He seems a tad erratic, echoing another thought of mine.

"With no continuum it falls apart."

I'm still blinking when, two seconds later, he pulls off a *coup de maître*.

"Einstein said so."

He's right: Einstein is always clear his theory works only if space is a continuum. His instincts tug him to the discontinuum but he can't get a handle on its math. He tells a colleague the continuum 'should be banned from the theory as a supplementary construction not justified by the essence of the problem, which corresponds to nothing "real". ' He says, 'But we still lack the mathematical structure unfortunately. How much I have already plagued myself in this way!' Even then this was an old idea. More than a hundred years before, Kronecker says: 'God made the integers; all the rest is the work of man.' Yet physics is still working with real numbers. His Beginning says that Kronecker and Einstein were both right. It may take another hundred years to calculate the consequence.

When we get back from lunch her Frank is going through his desk. I haven't seen him in an age but he just stares. Maybe he imagines I don't know what's going on. Of course I say no word of what *we're* doing. Just some pleasantry to smooth the awkwardness as he is on his way. He doesn't say goodbye but in my bones I know.